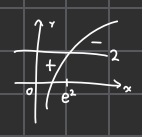
[1] $f(x) = x^{-\frac{1}{2}}\log x \qquad x>0$

1) $f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}\cdot\log x + x^{-\frac{1}{2}}\cdot x^{-1}$

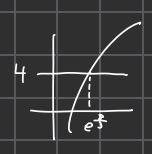
$= \frac{1}{2}x^{-\frac{3}{2}}(2-\log x)$



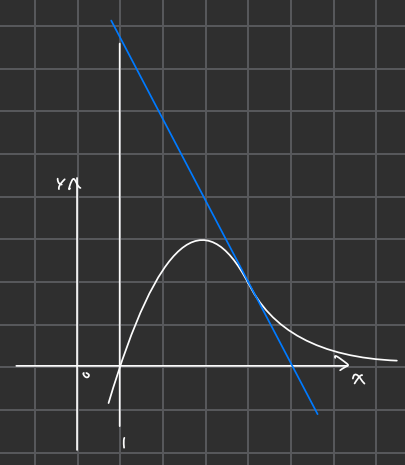
2) $f''(x) = -\frac{3}{2}\cdot\frac{1}{2}x^{-\frac{5}{2}}(2-\log x) + \frac{1}{2}x^{-\frac{3}{2}}\cdot -\frac{1}{x}$

$= \frac{1}{2}x^{-\frac{5}{2}}\left\{-\frac{3}{2}(2-\log x)-1\right\}$

$= \frac{1}{2}x^{-\frac{5}{2}}\left(\frac{3}{2}\log x - 4\right)$



$\left(e^{\frac{8}{3}},\ \frac{8}{3}e^{-\frac{4}{3}}\right)$ //



3) l について

$y - \frac{8}{3}e^{-\frac{4}{3}} = -\frac{1}{2}\cdot e^{-4}\left(2-\frac{8}{3}\right)\left(x - e^{\frac{8}{3}}\right)$ $\quad -\frac{2}{3}$

$y = +\frac{1}{3}\cdot e^{-4}\left(x - e^{\frac{8}{3}}\right) + \frac{8}{3}e^{-\frac{4}{3}}$

$\Leftrightarrow \quad y = +\frac{1}{3}e^{-4}x + \frac{8}{3}e^{-\frac{4}{3}} - \frac{1}{3}e^{-4}\cdot e^{\frac{8}{3}}$

$\Leftrightarrow \quad y = -\frac{1}{3}e^{-4}x + \frac{7}{3}e^{-\frac{4}{3}}$ //

$\int 2x^{-\frac{1}{2}}dx$

(4)

$S = \int_1^{e^{\frac{8}{3}}} -\frac{1}{3}e^{-4}x + \frac{7}{3}e^{-\frac{4}{3}} - x^{-\frac{1}{2}}\log x \quad dx$

$= \left[-\frac{1}{6}e^{-4}x^2 + 3e^{-\frac{4}{3}}x\right]_1^{e^{\frac{8}{3}}} - \int_1^{e^{\frac{8}{3}}} x^{-\frac{1}{2}}\log x\, dx$

$= \left\{\left(-\frac{1}{6}\cdot e^{-4}\cdot e^{\frac{16}{3}} + 3e^{-\frac{4}{3}}\cdot e^{\frac{8}{3}}\right) - \left(-\frac{1}{6}e^{-4} + 3e^{-\frac{4}{3}}\right)\right\} - \left\{\frac{13}{3}e^{\frac{4}{3}} + 4\right\}$

$= -\frac{1}{6}e^{\frac{4}{3}} + 3e^{\frac{4}{3}} + \frac{1}{6}e^{-4} - 3e^{-\frac{4}{3}} - \frac{13}{3}e^{\frac{4}{3}} - 4$

$= \left(-\frac{1}{6}+3-\frac{13}{3}\right)e^{\frac{4}{3}} + \frac{1}{6}e^{-4} - 3e^{-\frac{4}{3}} - 4$

$= -\frac{3}{2}e^{\frac{4}{3}} + \frac{1}{6}e^{-4} - 3e^{-\frac{4}{3}} - 4$

$\int_1^{e^{\frac{8}{3}}} x^{-\frac{1}{2}}\log x\, dx = \left[2x^{\frac{1}{2}}\log x\right]_1^{e^{\frac{8}{3}}} - \int 2\sqrt{x}\,\frac{1}{x}\,dx$

$= \left[2x^{\frac{1}{2}}\log x - 4x^{\frac{1}{2}}\right]_1^{e^{\frac{8}{3}}}$

$= \left(2e^{\frac{4}{3}}\cdot\frac{8}{3} - e^{\frac{4}{3}}\right) + 4$

$= \frac{13}{3}e^{\frac{4}{3}} + 4$

よって $S = -\frac{3}{2}$

$k = \frac{1}{6}$ //