

Ⅲ $A(3, 4, 3), B(-2, 2, 6)$

(1) 点 A, B の x, z 平面 における直線の式は

$$z = \frac{3-6}{3+2}(x-3)+3$$

$$= -\frac{3}{5}x + \frac{9}{5} + 3$$

$z=0$ の時

$$\frac{3}{5}x = \frac{9+5}{5}$$

$$x = 8$$

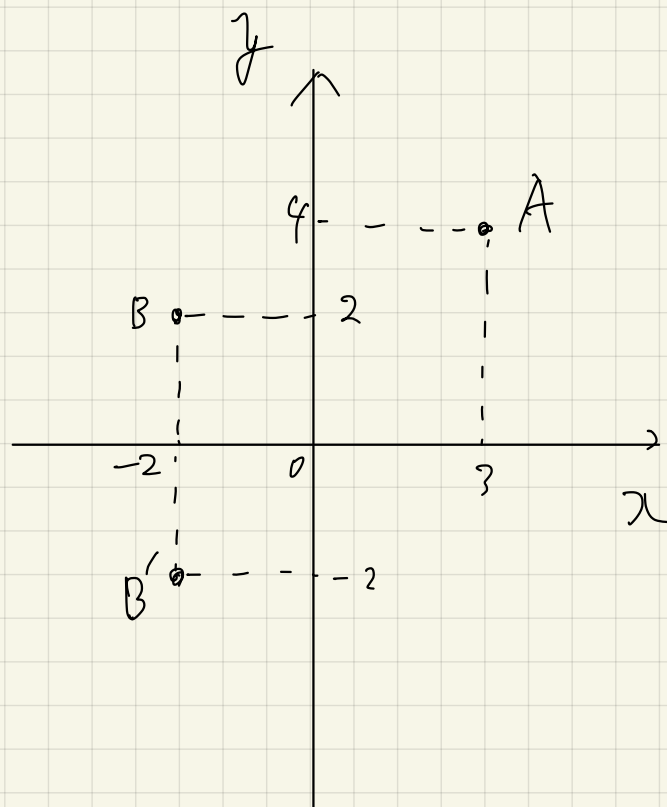
y, z 平面 における直線の式は、

$$z = \frac{3-6}{4-2}(y-4)+3$$

$$= -\frac{3}{2}y + 6 + 3$$

$z=0$ の時 $y = 6$

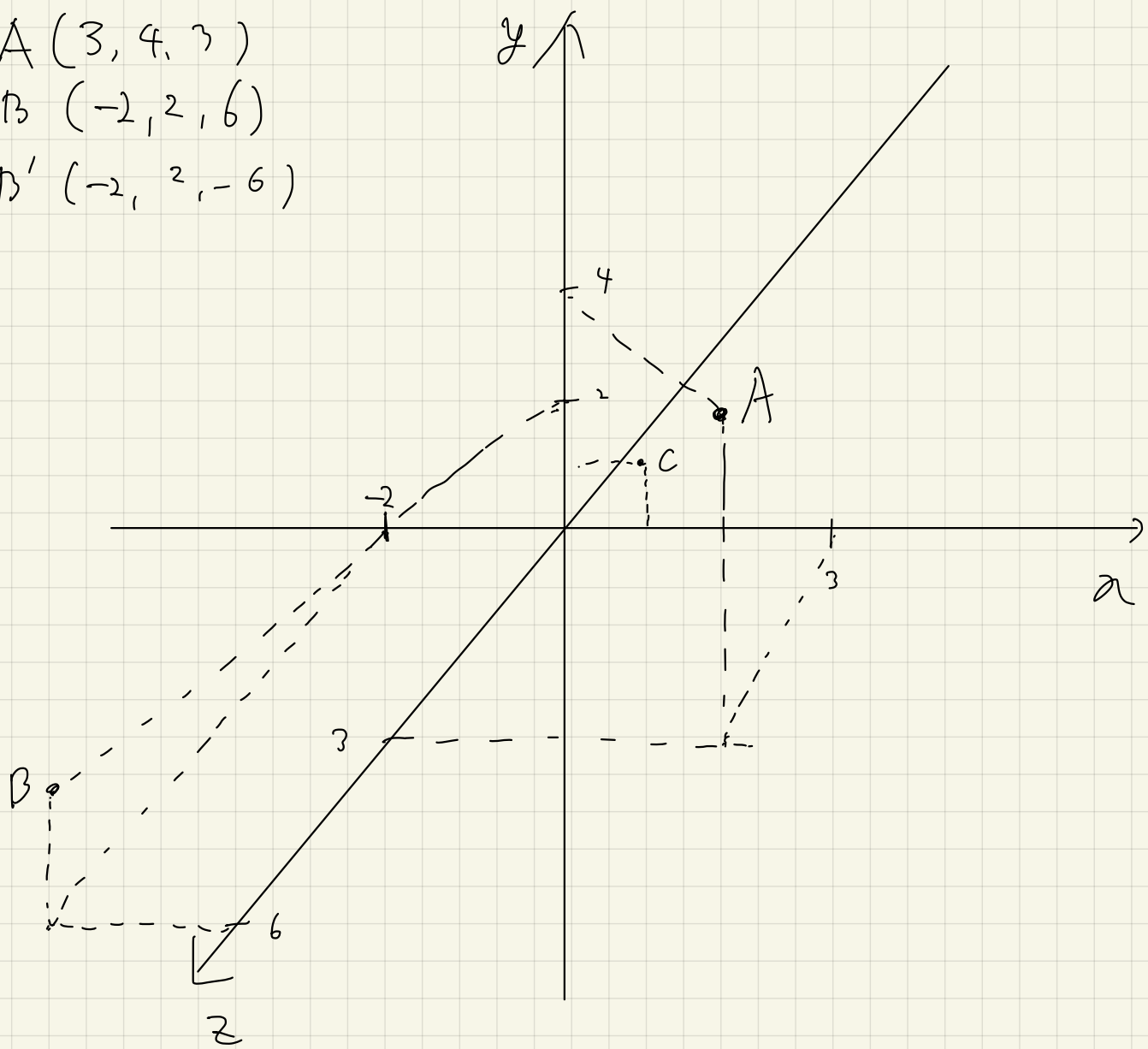
$\therefore \underline{(8, 6, 0)}$



$$A(3, 4, 3)$$

$$B(-2, 2, 6)$$

$$B'(-2, 2, -6)$$



$$B'(-2, 2, -6) \text{ とおこ}$$

$$(AC + CB)_{\min} = AC + CB'$$

$$= \sqrt{(3+2)^2 + (4-2)^2 + (3+6)^2}$$

$$= \sqrt{25 + 4 + 81}$$

$$= \sqrt{110} = -\frac{6}{5} + \frac{20}{5}$$

点 A と点 B' の式 (x, y, z) を

$$x \text{ y 平面} : y = \frac{4-2}{3+2}(x-3) + 4 = \frac{2}{5}x - \frac{6}{5} + 4$$

$$x \text{ z 平面} : x = \frac{3+2}{3+6}(z-3) + 3 = \frac{5}{9}z - \frac{5}{3} + 3$$

$$y \text{ z 平面} : x = \frac{4-2}{3+1}(z-3) + 4 = \frac{2}{9}z - \frac{2}{3} + 4$$

$$z = 0 \text{ の CF}$$

$$x = -\frac{5}{3} + 3 = \frac{4}{3}, \quad y = -\frac{2}{3} + 4 = \frac{10}{3}$$

$$\rightarrow C \left(\frac{4}{3}, \frac{10}{3}, 0 \right)$$

$$\overline{AC} = \begin{pmatrix} \frac{4}{3} \\ \frac{10}{3} \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{5}{3} \\ -\frac{2}{3} \\ -3 \end{pmatrix}$$

$$\overline{AB} = \begin{pmatrix} -2 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \\ 3 \end{pmatrix}$$

$$S = \frac{1}{2} \sqrt{|\overline{AC}|^2 |\overline{AB}|^2 - (\overline{AB} \cdot \overline{AC})^2}$$

$$|\overline{AC}|^2 = \frac{25}{9} + \frac{4}{9} + 9 = \frac{29+4+81}{9} = \frac{110}{9}$$

$$|\overline{AB}|^2 = 25 + 4 + 9 = 38$$

$$\overline{AC} \cdot \overline{AB} = \frac{25}{3} + \frac{4}{3} - 9 = \frac{29}{3} - \frac{27}{3} = \frac{2}{3}$$

$$\begin{aligned} S &= \frac{1}{2} \sqrt{\frac{110}{9} \times 38 - \frac{4}{9}} = \frac{1}{2} \sqrt{464} \\ &= \frac{1}{2} \times 4 \sqrt{29} = 2\sqrt{29} \end{aligned}$$

(3) 条件より点 $D(5, -1, d)$ とおける

$$\overrightarrow{AD} = \begin{pmatrix} 5 \\ -1 \\ d \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} = (2, -5, d-3)$$

$$\overrightarrow{AB} = (-5, -2, 3)$$

$$|\overrightarrow{AD}|^2 = 4 + 25 + (d-3)^2 = (d-3)^2 + 29$$

$$|\overrightarrow{AB}|^2 = 38$$

$$\overrightarrow{AD} \cdot \overrightarrow{AB} = -10 + 10 + 3d - 9 = 3d - 9$$

$$S = \frac{1}{2} \sqrt{38(d-3)^2 + 29 \cdot 38 - (3d-9)^2}$$

$$= \frac{1}{2} \sqrt{38(d-3)^2 - 9(d-3)^2 + 1102}$$

$$= \frac{1}{2} \sqrt{29(d-3)^2 + 1102}$$

$$F) \quad d=3 \text{ のとき } S \text{ は } \min \frac{\sqrt{1102}}{2} \text{ となる}$$