1

(1) $\vec{OA} + \sqrt{2}\,\vec{OB} + \sqrt{3}\,\vec{OC} = 0$

$$\vec{OA} + \sqrt{3}\,\vec{OC} = -\sqrt{2}\,\vec{OB}$$

$$|\vec{OA}|^2 + 2\sqrt{3}\,\vec{OA}\cdot\vec{OC} + 3|\vec{OC}|^2 = 2|\vec{OB}|^2$$

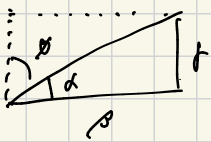
$$1 + 2\sqrt{3}\,\vec{OA}\cdot\vec{OC} + 3 = 2$$

$$\vec{OA}\cdot\vec{OC} = -\frac{1}{\sqrt{3}}$$ //

(2) $\tan\frac{\pi}{10}\tan\frac{2}{5}\pi = \tan\frac{\pi}{10} \times \tan\left(\frac{\pi}{2}-\frac{\pi}{10}\right)$

$$= \tan\frac{\pi}{10} \times \frac{1}{\tan\frac{\pi}{10}} = 1$$ //

(pf)



$$\tan\alpha = \frac{\gamma}{\beta}$$

$$\tan\theta = \tan\left(\frac{\pi}{2}-\alpha\right)$$

$$= \frac{\beta}{\gamma}$$

$$= \frac{1}{\tan\alpha}$$

よって $\tan\left(\frac{\pi}{2}-\theta\right) = \frac{1}{\tan\theta}$

(3) $S_n = 3^n + 2a_n$

$a_1 = 3 + 2a_1$

$a_1 = -3$

$S_{n+1} = 3^{n+1} + 2a_{n+1}$

$-)\ S_n = 3^n + 2a_n$

$a_{n+1} - a_n = 2 \cdot 3^n + 2(a_{n+1} - a_n)$

$a_{n+1} = a_n - 2 \cdot 3^n$

(3^{n+1}로 나누기)

$$\frac{a_{n+1}}{3^{n+1}} = \frac{a_n}{3^n} - \frac{2}{3}$$

$b_{n+1} = b_n - \frac{2}{3}$ $(b_1 = -1)$

$b_n = -\frac{2}{3}n - \frac{1}{3}$

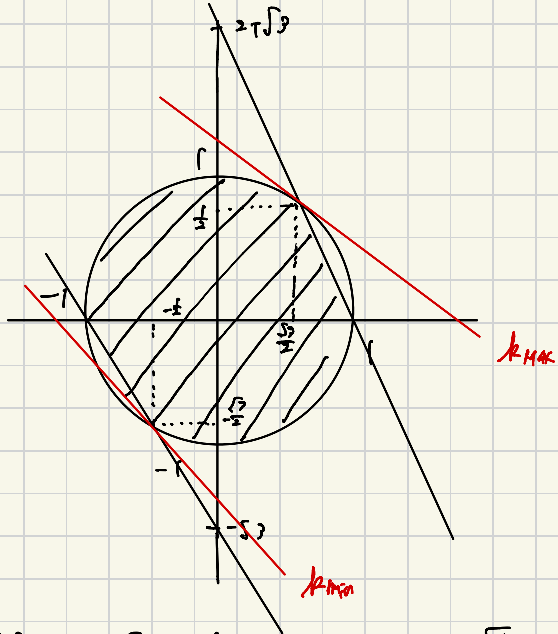
(4) $y = a_1 x + b_1$ 로 치환

$$y = \frac{\frac{1}{2} - 0}{\frac{\sqrt{3}}{2} - 1}(x - 1) + 0$$

$$= -(2 + \sqrt{3})x + 2 + \sqrt{3}$$

$y = a_2 x + b_2$ を代入

$y = -\sqrt{3}x - \sqrt{3}$



$x+y=k$ とおいて 線形計画法（接点のx座標の$\frac{\sqrt{3}}{2}$との大小で場合分け）

最大値 $\frac{1+\sqrt{3}}{2}$ 　最小値 $-\frac{1+\sqrt{3}}{2}$

(5)

$\cos\theta_1 - \cos\theta_2 = 0$

$\cos^2\theta_1 - 2\cos\theta_1\cos\theta_2 + \cos^2\theta_2 = 0$ …①

$\sin\theta_1 - \sin\theta_2 = 1$

$\sin^2\theta_1 - 2\sin\theta_1\sin\theta_2 + \sin^2\theta_2 = 1$ …②

①+②より $2 - 2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) = 1$

$\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 = \frac{1}{2}$

$\cos(\theta_1 - \theta_2) = \frac{1}{2}$ （加法定理の逆）

$\theta_2 - \theta_1 = \frac{\pi}{3},\ \frac{5}{3}\pi$

$\cos\theta_1 - \cos\theta_2 = 0$ より

$(\theta_1, \theta_2) = \left(\frac{5}{6}\pi, \frac{7}{6}\pi\right)\left(\frac{\pi}{6}, \frac{11}{6}\pi\right)$

[2] (1) n回引いてすべてハズれる確率は

$$\left(\frac{k-1}{k}\right)^n$$

よって1回以上当たる確率は

$$1 - \left(\frac{k-1}{k}\right)^n$$

よって $1 - \left(\frac{k-1}{k}\right)^n \geqq a$ …(※)

$k=3$ $n=2$ $a=\frac{5}{9}$ のとき

(※)の(左辺) $= 1 - \left(\frac{3-1}{3}\right)^2$

$= 1 - \frac{4}{9} = \frac{5}{9}$ より成立

■

(2)

$$1-\left(\frac{k-1}{k}\right)^n \geqq a$$

$$\left(\frac{k-1}{k}\right)^n \leqq 1-a$$

底が $\frac{k-1}{k}$ の対数をとる $\left(0<\frac{k-1}{k}<1 \text{ かつ } 1-a>0\right)$

$$n \geqq \log_{\frac{k-1}{k}}(1-a)$$

(3) (2)の式に $k=5$, $a=0.99$ を代入する

$$n \geqq \log_{\frac{4}{5}}(0.01)$$

$$= \frac{\log_5 (5\times 2)^{-2}}{\log_5 \frac{4}{5}}$$

$$= \frac{-2(1+\log_5 2)}{2\log_5 2 - 1}$$

$$= \frac{2(1+\log_5 2)}{1-2\log_5 2}$$

$$= \frac{2.86}{1-0.86}$$

$$\fallingdotseq 20.42\cdots$$

よって $n=21$

[3] (1) $f(x) = \dfrac{1}{1+x^2}$ とおく

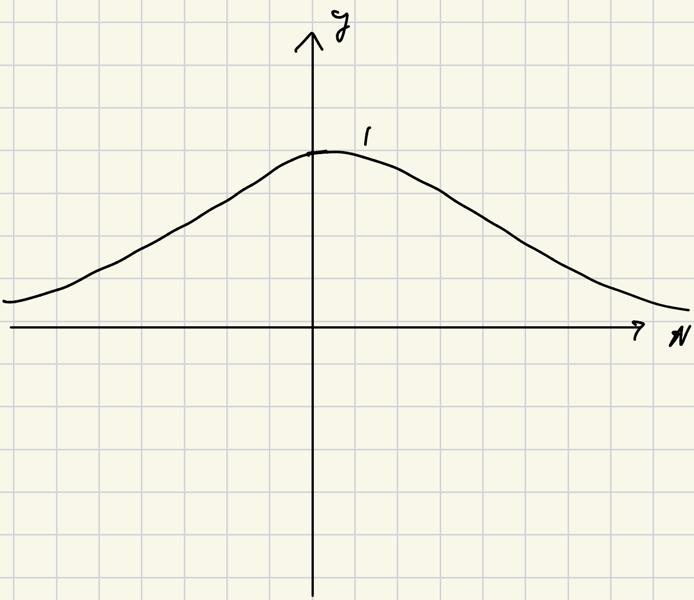
$$f'(x) = \frac{-2x}{(1+x^2)^2}$$

$$f''(x) = \frac{-2(1+x^2)^2 + 2x \times 2(1+x^2) \times 2x}{(1+x^2)^4}$$

$$= \frac{-2 - 2x^2 + 8x^2}{(1+x)^3}$$

$$= \frac{2(3x^2 - 1)}{(1+x^2)^3}$$

x	$\cdots$	$-\frac{1}{\sqrt{3}}$	$\cdots$	0	$\cdots$	$\frac{1}{\sqrt{3}}$	
$f'(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$
$f''(x)$	$+$	0	$-$	$-$	$-$	0	$+$
$f(x)$	↗	$\frac{3}{4}$	↗	1	↘	$\frac{3}{4}$	↘



(2) $I = \int_{-1}^{1} \frac{1}{1+x^2}\,dx$

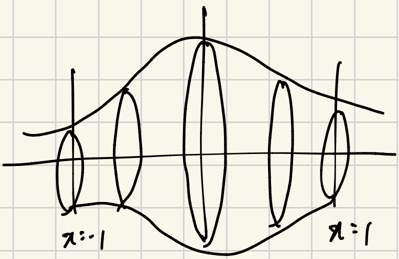
偶関数なので $I = 2\int_{0}^{1} \frac{1}{1+x^2}\,dx$

$x = \tan\theta$ と置換して

⋮

$I = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

(3)



$V = 2\pi \int_{0}^{1} \frac{1}{(1+x^2)^2}\,dx$

$x = \tan\theta$

$\frac{dx}{d\theta} = \frac{1}{\cos^2\theta}$

$dx = \frac{1}{\cos^2\theta}\,d\theta$

x	$0 \to 1$
θ	$0 \to \frac{\pi}{4}$

$V = 2\pi \int_{0}^{\frac{\pi}{4}} \frac{1}{(1+\tan^2\theta)^2} \times \frac{1}{\cos^2\theta}\,d\theta$

$$= 2\pi \int_0^{\frac{\pi}{4}} \cos^2\theta \, d\theta$$

$$= \pi \int_0^{\frac{\pi}{4}} (1 + \cos 2\theta) d\theta$$

$$= \pi \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{4}}$$

$$= \pi \left(\frac{\pi}{4} + \frac{1}{2} \right) = \frac{\pi}{4} (\pi + 2)$$